Technical Notes

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Asymmetric Stress Analysis of Axisymmetric Solids with Anisotropic Material Properties

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I. Introduction

THE solid of revolution stress-analysis computer code modifications recently were presented[1] to include a rectangularly orthotropic material model. This particular material model has considerable interest in the application of carbon/carbon composite materials for re-entry vehicle nosetips. Recently another breed of composite materials (referred to as 4-D and 7-D weave composites) has aroused the interest of nosetip designers. The development of these advanced composites is motivated toward obtaining more uniform strength and thermal expansion properties while minimizing the occurrence of unsupported yarn bundles on the ablating surface of nosetips. Unfortunately, these composite materials cannot be modeled correctly with the present generation of the solid of revolution stress-analysis codes, since these composites are neither transversely isotropic, polar orthotropic, nor rectangularly orthotropic, but rather anisotropic. Thus, at the expense of meeting nosetip design requirements, an anisotropic material results, with its inherent three-dimensional stress-analysis requirements.

An example of these advanced composites that would require a three-dimensional stress analysis method is a 4-D weave, which consists of carbon-reinforcing fibers bundled in four directions (see Fig. 1). The woven preform is then impregnated with a suitable matrix material and processed to achieve the designed properties. For analysis purposes, these 4-D composite materials usually are considered to be homogeneous anisotropic, it being common engineering practice to ignore the inhomogeneity or discreteness of the reinforcements. To model the macroscopic behavior at present, a 3-D code is required, with its inherent high computer costs. The purpose of this Note is to present modifications that must be made to a 2-D computer code to enable the analysis of 3-D anisotropic materials fabricated into axisymmetric shapes and loaded axi- or asymmetrically. Specifically, the modification is presented for the ASAAS[2] (asymmetric stress analysis of axisymmetric solids) structural analysis code. Inasmuch as the ASAAS code modifications for an anisotropic material parallel those for a rectangularly orthotropic material, this Note reflects only the significant changes from Ref. 1; maximum use will be made of these results. One can obtain more details on the axisymmetric solid stress analysis procedure by consulting Refs. 2-4.

II. ASAAS Code Modifications

The ASAAS code solves the thermostructural response problem for solids-of-revolution subjected to asymmetric surface traction and temperature distributions. Since the variation of material properties with temperature is included within the formulation, asymmetric material property capability is inherent in the program. The analysis is formulated in polar coordinates; thus all variations of loads, displacements, and material properties in the circumferential direction are conveniently represented by a truncated Fourier series. If, however, there is no circumferential variation of temperatures, the material properties as well as the element stiffness matrices of anisotropic materials are still dependent on the circumferential coordinate θ. The computer code modifications to accommodate an anisotropic material model are confined principally to the elasticity matrix, stiffness matrix, and thermal load vector.

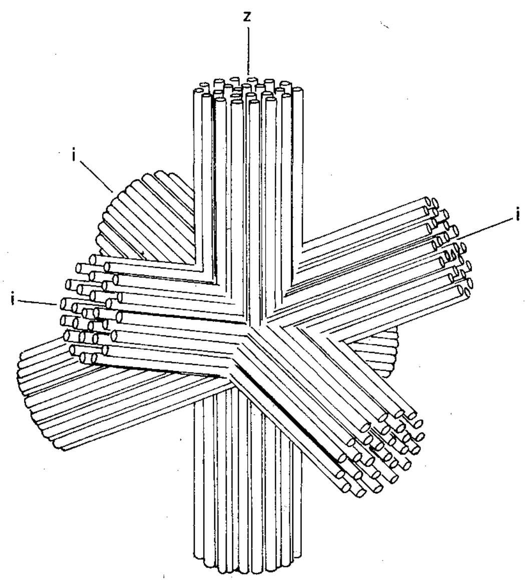

Fig. 1 4-D carbon/carbon composite.

Elasticity Matrix

Since a fundamental consideration of the present discussion is to incorporate the representation of a material into equations that are inherently polar, it is necessary to consider the transformation of the element elasticity matrix from rectangular to polar coordinates. A typical elastic constant E^*_{ij} in polar coordinates can be expressed in terms of the Cartesian elastic constants E_{ij} at any angle θ. Specifically, the detailed relationships between the polar and Cartesian terms are

$$E^*_{11} = \tfrac{1}{8}[4E_{55}+2E_{13}+3(E_{11}+E_{33})] + \tfrac{1}{2}[E_{11}-E_{33}]\cos 2\theta + \tfrac{1}{8}[E_{11}+E_{33}-4E_{55}-2E_{13}]\cos 4\theta + [E_{15}+E_{35}]\sin 2\theta + \tfrac{1}{2}[E_{15}-E_{35}]\sin 4\theta$$

$$E^*_{12} = \tfrac{1}{2}[E_{12}+E_{23}] + \tfrac{1}{2}[E_{12}-E_{23}]\cos 2\theta + E_{25}\sin 2\theta$$

$$E^*_{13} = \tfrac{1}{8}[E_{11}+E_{33}+6E_{13}-4E_{55}] - \tfrac{1}{8}[E_{11}+E_{33}-2E_{13} - 4E_{55}]\cos 4\theta - \tfrac{1}{2}[E_{15}-E_{35}]\sin 4\theta$$

$$E^*_{14} = \tfrac{1}{4}[3E_{14}+E_{34}+2E_{56}]\cos\theta + \tfrac{1}{4}[E_{14}-E_{34}-2E_{56}]\cos 3\theta + \tfrac{1}{4}[E_{16}+3E_{36}+2E_{54}]\sin\theta + \tfrac{1}{4}[E_{16}-E_{36}+2E_{54}]\sin 3\theta$$

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$$E^*_{15} = \tfrac{1}{2}[E_{15}+E_{35}]\cos 2\theta + \tfrac{1}{2}[E_{15}-E_{35}]\cos 4\theta + \tfrac{1}{4}[E_{33}-E_{11}]\sin 2\theta - \tfrac{1}{8}[E_{11}+E_{33}-4E_{55}-2E_{13}]\sin 4\theta$$

$$E^*_{16} = \tfrac{1}{4}[3E_{16}+E_{36}-2E_{54}]\cos\theta + \tfrac{1}{4}[E_{16}-E_{36}+2E_{54}]\cos 3\theta - \tfrac{1}{4}[E_{14}+3E_{34}-2E_{56}]\sin\theta - \tfrac{1}{4}[E_{14}-E_{34}-2E_{56}]\sin 3\theta$$

$$E^*_{22} = E_{22} \qquad E^*_{23} = \tfrac{1}{2}[E_{12}+E_{23}] + \tfrac{1}{2}[E_{23}-E_{12}]\cos 2\theta - E_{25}\sin 2\theta \qquad E^*_{24} = E_{24}\cos\theta + E_{26}\sin\theta$$

$$E^*_{25} = E_{25}\cos 2\theta + \tfrac{1}{2}[E_{23}-E_{12}]\sin 2\theta \qquad E^*_{26} = E_{26}\cos\theta - E_{24}\sin\theta$$

$$E^*_{33} = \tfrac{1}{8}[4E_{55}+2E_{13}+3(E_{11}+E_{33})] + \tfrac{1}{2}[E_{33}-E_{11}]\cos 2\theta + \tfrac{1}{8}[E_{11}+E_{33}-4E_{55}-2E_{13}]\cos 4\theta - [E_{15}+E_{35}]\sin 2\theta + \tfrac{1}{2}[E_{15}-E_{35}]\sin 4\theta$$

$$E^*_{34} = \tfrac{1}{4}[E_{14}+3E_{34}-2E_{56}]\cos\theta - \tfrac{1}{4}[E_{14}-E_{34}-2E_{56}]\cos 3\theta + \tfrac{1}{4}[3E_{16}+E_{36}-2E_{54}]\sin\theta - \tfrac{1}{4}[E_{16}-E_{36}+2E_{54}]\sin 3\theta$$

$$E^*_{35} = \tfrac{1}{2}[E_{15}+E_{35}]\cos 2\theta - \tfrac{1}{2}[E_{15}-E_{35}]\cos 4\theta + \tfrac{1}{4}[E_{33}-E_{11}]\sin 2\theta + \tfrac{1}{8}[E_{11}+E_{33}-4E_{55}-2E_{13}]\sin 4\theta$$

$$E^*_{36} = \tfrac{1}{4}[E_{16}+3E_{36}+2E_{54}]\cos\theta - \tfrac{1}{4}[E_{16}-E_{36}+2E_{54}]\cos 3\theta - \tfrac{1}{4}[3E_{14}+E_{34}+2E_{56}]\sin\theta + \tfrac{1}{4}[E_{14}-E_{34}-2E_{56}]\sin 3\theta$$

$$E^*_{44} = \tfrac{1}{2}[E_{44}+E_{66}] + \tfrac{1}{2}[E_{44}-E_{66}]\cos 2\theta + E_{46}\sin 2\theta$$

$$E^*_{45} = -\tfrac{1}{4}[E_{16}-E_{36}-2E_{45}]\cos\theta + \tfrac{1}{4}[E_{16}-E_{36}+2E_{45}]\cos 3\theta - \tfrac{1}{4}[E_{14}-E_{34}+2E_{56}]\sin\theta - \tfrac{1}{4}[E_{14}-E_{34}-2E_{56}]\sin 3\theta$$

$$E^*_{46} = E_{46}\cos 2\theta + \tfrac{1}{2}[E_{66}-E_{44}]\sin 2\theta \qquad E^*_{55} = \tfrac{1}{8}[E_{11}+E_{33}-2E_{13}+4E_{55}] - \tfrac{1}{8}[E_{11}+E_{33}-2E_{13}-4E_{55}]\cos 4\theta - \tfrac{1}{2}[E_{15}-E_{35}]\sin 4\theta$$

$$E^*_{56} = \tfrac{1}{4}[E_{14}-E_{34}+2E_{56}]\cos\theta - \tfrac{1}{4}[E_{14}-E_{34}-2E_{56}]\cos 3\theta - \tfrac{1}{4}[E_{16}-E_{36}-2E_{54}]\sin\theta - \tfrac{1}{4}[E_{16}-E_{36}+2E_{54}]\sin 3\theta$$

$$E^*_{66} = \tfrac{1}{2}[E_{44}+E_{66}] + \tfrac{1}{2}[E_{66}-E_{44}]\cos 2\theta + E_{46}\sin 2\theta \qquad (1)$$

Stiffness Matrix

Inasmuch as the components of the element strain vector also are assumed to vary circumferentially, the volume integral for the stiffness matrix may be represented by

$$K = \sum_{m_\ell=0}^{M_\ell} \sum_{m_d=0}^{M_d} h^T \phi_{m_\ell, m_d} h = h^T \left\{ \int_{A_i} \int_0^{2\pi} g^T_{m_\ell} E_{m_m} g_{m_d} r\,\mathrm{d}\theta\,\mathrm{d}A \right\} h \qquad (2)$$

where the matrices h and g relate the element strains to nodal displacements with matrix h constant for the element volume and matrix g variable over the element volume. The subscripts m_ℓ, m_d, m_m refer to load, displacement, and material harmonics, respectively. The 9×9 matrix, h, is an assemblage of geometrical constants and is therefore the same for a solid-of-revolution constructed of an isotropic, polar orthotropic, rectangularly orthotropic, or anisotropic material. The 9×9 matrix, ϕ_{m_ℓ, m_d}, on the other hand, is altered depending upon the material used. Thus, changes to the element stiffness matrix to account for an anisotropic material are the direct result of modifications to the matrix $\phi_{m_\ell m_d}$. These modifications are the result of incorporating the 6×6 matrix of material property Fourier coefficients, E_{m_m}, in the derivation of $\phi_{m_\ell m_d}$. The coefficients of the $\phi_{m_\ell m_d}$ matrix are given in the form $\phi_{ij} = \phi_{ij}$ (Ref. 1) $+ \phi^*_{ij}$ where ϕ^*_{ij} contains the additional terms arising from the extension presented in this Note. Thus,

$$\phi^*_{31} = [\bar C_{43} - m_d \bar C_{45}]\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{41} = [-m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int (1/r)\,\mathrm{d}r\mathrm{d}z \qquad \phi^*_{51} = [\bar C_{43} - m_d \bar C_{45} - m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{61} = [-m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int (z/r)\,\mathrm{d}r\mathrm{d}z \qquad \phi^*_{91} = [\bar C_{63} - m_d \bar C_{65}]\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{32} = [\bar C_{41} + \bar C_{43} - m_d \bar C_{45}]\int r\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{42} = [-m_\ell \bar C_{61} - m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{52} = [\bar C_{41} + \bar C_{43} - m_d \bar C_{45} - m_\ell \bar C_{61} - m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int r\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{62} = [-m_\ell \bar C_{61} - m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{92} = [\bar C_{61} + \bar C_{63} - m_d \bar C_{65}]\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{13} = [\bar C_{34} - m_\ell \bar C_{54}]\int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{23} = [\bar C_{14} + \bar C_{34} - m_\ell \bar C_{54}]\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{33} = [\bar C_{34} + \bar C_{43} - m_d \bar C_{45} - m_\ell \bar C_{54}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{43} = [-m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int (z/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{53} = [\bar C_{43} - m_d \bar C_{45} - m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{63} = \bar C_{24}\int r\mathrm{d}r\mathrm{d}z + [-m_\ell \bar C_{63} + m_\ell m_d \bar C_{65}]\int (z^2/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{73} = [m_\ell \bar C_{34} - \bar C_{54}]\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{83} = m_\ell \bar C_{34}\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{93} = [m_\ell \bar C_{34} - \bar C_{54} + \bar C_{63} - m_d \bar C_{65}]\int z\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{14} = [-m_d \bar C_{36} + m_\ell m_d \bar C_{56}]\int (1/r)\,\mathrm{d}r\mathrm{d}z \qquad \phi^*_{24} = [-m_d \bar C_{16} - m_d \bar C_{36} + m_\ell m_d \bar C_{56}]\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{34} = [-m_d \bar C_{36} + m_\ell m_d \bar C_{56}]\int (z/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{64} = -m_d \bar C_{26}\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{74} = [-m_\ell m_d \bar C_{36} + m_d \bar C_{56}]\int (1/r)\,\mathrm{d}r\mathrm{d}z \qquad \phi^*_{84} = -m_\ell m_d \bar C_{36}\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{94} = [-m_\ell m_d \bar C_{36} + m_d \bar C_{56}]\int (z/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{15} = [\bar C_{34} - m_d \bar C_{36} - m_\ell \bar C_{54} + m_\ell m_d \bar C_{56}]\int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{25} = [\bar C_{14} - m_d \bar C_{16} + \bar C_{34} - m_d \bar C_{36} - m_\ell \bar C_{54} + m_\ell m_d \bar C_{56}]\int r\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{35} = [\bar C_{34} - m_d \bar C_{36} - m_\ell \bar C_{54} + m_\ell m_d \bar C_{56}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{65} = [\bar C_{24} - m_d \bar C_{26}]\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{75} = [m_\ell \bar C_{34} - m_\ell m_d \bar C_{36} - \bar C_{54} + m_d \bar C_{56}]\int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{85} = [m_\ell \bar C_{34} - m_\ell m_d \bar C_{36}]\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{95} = [m_\ell \bar C_{34} - m_\ell m_d \bar C_{36} - \bar C_{54} + m_d \bar C_{56}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{16} = [-m_d \bar C_{36} + m_\ell m_d \bar C_{56}]\int (z/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{26} = [-m_d \bar C_{16} - m_d \bar C_{36} + m_\ell m_d \bar C_{56}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{36} = [-m_d \bar C_{36} + m_\ell m_d \bar C_{56}]\int (z^2/r)\,\mathrm{d}r\mathrm{d}z + \bar C_{42}\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{46} = -m_\ell \bar C_{62}\int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{56} = [\bar C_{42} - m_\ell \bar C_{62}]\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{66} = [-m_d \bar C_{26} - m_\ell \bar C_{62}]\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{76} = [-m_\ell m_d \bar C_{36} + m_d \bar C_{56}]\int (z/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{86} = -m_\ell m_d \bar C_{36}\int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{96} = [-m_\ell m_d \bar C_{36} + m_d \bar C_{56}]\int (z^2/r)\,\mathrm{d}r\mathrm{d}z + \bar C_{62}\int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{37} = [m_d \bar C_{43} - \bar C_{45}]\int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{47} = [-m_\ell m_d \bar{C}_{63} + m_\ell \bar{C}_{65}] \int (1/r)\,\mathrm{d}r\mathrm{d}z \qquad \phi^*_{57} = [m_d \bar{C}_{43} - \bar{C}_{45} - m_\ell m_d \bar{C}_{63} + m_\ell \bar{C}_{65}] \int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{67} = [-m_\ell m_d \bar{C}_{63} + m_\ell \bar{C}_{65}] \int (z/r)\,\mathrm{d}r\mathrm{d}z \qquad \phi^*_{97} = [m_d \bar{C}_{63} - \bar{C}_{65}] \int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{38} = m_d \bar{C}_{43} \int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{48} = -m_\ell m_d \bar{C}_{63} \int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{58} = [m_d \bar{C}_{43} - m_\ell m_d \bar{C}_{63}] \int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{68} = -m_\ell m_d \bar{C}_{63} \int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{98} = m_d \bar{C}_{63} \int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{19} = [\bar{C}_{36} - m_\ell \bar{C}_{56}] \int \mathrm{d}r\mathrm{d}z$$

$$\phi^*_{29} = [\bar{C}_{16} + \bar{C}_{36} - m_\ell \bar{C}_{56}] \int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{39} = [\bar{C}_{36} + m_d \bar{C}_{43} - \bar{C}_{45} - m_\ell \bar{C}_{56}] \int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{49} = [-m_\ell m_d \bar{C}_{63} + m_\ell \bar{C}_{65}] \int (z/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{59} = [m_d \bar{C}_{43} - \bar{C}_{45} - m_\ell m_d \bar{C}_{63} + m_\ell \bar{C}_{65}] \int z\mathrm{d}r\mathrm{d}z \qquad \phi^*_{69} = \bar{C}_{26} \int r\mathrm{d}r\mathrm{d}z + [-m_\ell m_d \bar{C}_{63} + m_\ell \bar{C}_{65}] \int (z^2/r)\,\mathrm{d}r\mathrm{d}z$$

$$\phi^*_{79} = [m_\ell \bar{C}_{36} - \bar{C}_{56}] \int \mathrm{d}r\mathrm{d}z \qquad \phi^*_{89} = m_\ell \bar{C}_{36} \int r\mathrm{d}r\mathrm{d}z \qquad \phi^*_{99} = [m_\ell \bar{C}_{36} - \bar{C}_{56} + m_d \bar{C}_{63} - \bar{C}_{65}] \int z\mathrm{d}r\mathrm{d}z \tag{3}$$

The compact elastic coefficients $\bar{C}_{ij}$ are defined in the Appendix of Ref. 1.

Thermal Load Vector

The changes to the thermal load vector for an anisotropic material are considerably simpler than the modifications to the stiffness matrix. Briefly, the thermal stress vector, which is used to compute the thermal loads, is derived from the product of the elasticity matrix and the thermal strain vector. The thermal stress in polar coordinates due to complete restraint of thermal expansion is

$$\sigma_{R\theta Z} = E_{R\theta Z} \alpha_{R\theta Z} \Delta t \tag{4}$$

where

$$\alpha_{R\theta Z} = \begin{Bmatrix} \tfrac{1}{2}(\alpha_X + \alpha_Y) + \tfrac{1}{2}(\alpha_X - \alpha_Y)\cos 2\theta + \alpha_{XY}\sin 2\theta \\ \alpha_Z \\ \tfrac{1}{2}(\alpha_X + \alpha_Y) + \tfrac{1}{2}(\alpha_Y - \alpha_X)\cos 2\theta - \alpha_{XY}\sin 2\theta \\ \alpha_{XZ}\cos\theta + \alpha_{ZY}\sin\theta \\ (\alpha_Y - \alpha_X)\sin 2\theta \\ -\alpha_{XZ}\sin\theta + \alpha_{ZY}\cos\theta \end{Bmatrix} \tag{5}$$

III. Conclusions

This paper presents the modifications that must be made to the ASAAS finite-element code to enable the analysis of anisotropic materials fabricated into axisymmetric shapes and loaded axi- or asymmetrically. These modifications represent a significant increase in material modeling capability; i.e., any material that can be characterized by 21 elastic constants can be analyzed accurately. Furthermore, it enables one to analyze these solids-of-revolution with a pseudo-3-D computer code rather than having to resort to the more costly 3-D finite-element model.

Acknowledgments

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Blast Wave Analysis for Detonation Propulsion

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Introduction

A PROPULSION system based on chemical detonation rather than deflagration has been studied for the last several years (see, for example, Refs. 1-3). Larger specific impulse is obtainable from this system in high-density, high-pressure environments than from conventional rockets with propellant deflagration.

The performance of such a system was measured with the experimental system shown in Fig. 1, Ref. 1, in several different types of gases at different pressures. A conical nozzle (15.9 cm long) with a half angle, $\theta = 10$ deg, and a flat endwall (1.11 cm radius) was used. A small amount of explosive "detasheet" was placed flat against the nozzle endwall and detonated by a microdetonator. Results verify the advantages of the system over conventional rockets at high ambient pressures regardless of the type of gas.

A one-dimensional gasdynamic analysis of the system was carried out to explain qualitatively the general trend of the performance of the system at various conditions.[1] However, quantitative comparisons between this analysis and experiments were hindered by the inability to correctly calculate the blast wave coming out of the nozzle and the subsequent complex flow developing in and around the nozzle.

Two-Dimensional Computations

Reported herein is a two-dimensional inviscid calculation of this flowfield in the 69 bars, nitrogen environment. The Eulerian computer code DORF[4] was used with the explosive product gas assumed to follow the JWL equation of state. For details of computation, refer to Kim and Johnson.[3]

Figure 1 shows shock-front speed obtained from the local slope of the trajectory curves. Near the endwall, the shock speed could not be reliably determined because the shock was in the process of being formed and thus did not show a clear front. The experimentally obtained shock velocity[5] is compared in the figure as an asterisk.

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